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Effect of free-stream fluctuations on laminar forced convection from a straight tube

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Abstract—The effect of free-stream fluctuations on forced convection heat transfer from a straight tube of circular cross-section is investigated. The tube is assumed to have an isothermal surface and is placed in an unsteady but uniform cross stream. The free-stream fluctuations are represented by periodic (sinusoidal) fluctuations superimposed on the average stream velocity. The resulting unsteady velocity and thermal fields are obtained by solving the conservation equations of mass, momentum and energy. The main parameters involved are Reynolds number, Strouhal number, Prandtl number and the relative amplitude of fluctuations. The paper focuses on the effects of the amplitude and frequency of free-stream fluctuations on the local and average Nusselt numbers while keeping Prandtl number unchanged ($Pr = 0.7$). The average flow Reynolds number ranges from 50 to 500, the Strouhal number ranges from $\pi/4$ to π and the amplitude ranges from 20 to 50% of the free-stream average velocity. The study revealed that the effect of fluctuations on the time-average Nusselt number becomes more pronounced with increasing Reynolds number. It also revealed that the rate of heat transfer increases with the increase of the amplitude but decreases with the increase of frequency. The time-average Nusselt number is tabulated for all cases studied and the details of flow and thermal fields are presented in the form of local Nusselt number and surface vorticity distributions as well as streamline and temperature contours for some selected cases. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The effect of free-stream fluctuations on heat transfer from tubes, cylinders or wires has attracted the interest of many researchers during the last two decades because of the related engineering applications in heat exchangers, cooling of electronic equipment, hot-wire anemometry and many others. Such fluctuations may be externally imposed or naturally induced as in the case of vortex shedding from an upstream tube in a heat exchanger or a wire vibrating in a uniform stream. In all cases, an unsteady free stream results in an unsteady heat transfer. This study focuses on the effect of free-stream fluctuations/pulsations on the velocity and thermal boundary layers over a tube of circular cross-section.

One of the early experimental studies on the effect of oscillations on heat transfer from a cylinder in a cross-flow was that by Zijnen [1] who found that in-line oscillations results in a maximum decrease of 4.3% in the heat transfer coefficient for low Reynolds number flows ($Re < 5$). Sreenivasan and Ramachandran [2] conducted similar experiments but focused on oscillations in a direction normal to the main stream. The study revealed that imposing vibrational velocities up to 20% of flow velocity results in no significant change in heat transfer. A correlation for the average Nusselt number was reported for the range $Re = 2500$ to 15 000 in the absence of oscillations. The experimental study conducted by Saxena and Laird [3] focused on the effect of transverse oscillations on local heat transfer from a vertical cylinder placed in a cross-flow at a Reynolds number of 3500. The frequency of oscillations varied between 0.4 and 1.2 Hz while the amplitude varied from 0.89 to 1.99 times the cylinder diameter. It was found that the amplitude and frequency have equal contribution to the increase of the local heat transfer. The increase on the back side of the cylinder was about 15% more than that on the front side. In comparison with the case of no oscillations, the increase in the local heat transfer coefficient was up to 60% occurring at the back of the cylinder for the largest values of amplitude and frequency. Leung *et al.* [4] carried out a similar study, but focused on the effect of in-line vibrations in the Reynolds number range up to 5×10^4 and for only two amplitudes and two frequencies. The results showed that heat transfer was enhanced at both frequencies with further enhancement by increasing the amplitude in the range $Re < 1.5 \times 10^4$. However, at higher Re values ($Re > 2.5 \times 10^4$) the heat transfer rates were suppressed by vibrational effects. A new correlation method for the effect of vibration on forced convection from a sphere, a cylinder, and a tube of a square cross-section was presented in the work by Takahashi and Endoh [5]. The correlation relates the increase in heat transfer to the time-averaged energy dissipation and was based on a large amount of data obtained experimentally for three shapes. The cor-

NOMENCLATURE

a	tube radius
f_n, g_n	functions defined in equation (5)
h	heat transfer coefficient
H_o, H_n	functions defined in equation (5)
Nu, $\overline{Nu}$	local and average Nusselt numbers defined in Equation (9)
$\overline{\overline{Nu}}$	time-average value of $\overline{Nu}$
Pr	Prandtl number
Pe	Peclet number, $RePr$
Re	Reynolds number, $2aU_o/\nu$
S	Strouhal number, $a\alpha/U_o$
t	dimensionless time
T	fluid temperature
U	free-stream velocity
U_o	average velocity of free stream
v_ξ, v_θ	dimensionless ξ and θ velocity components.

Greek symbols

α	frequency of fluctuations (rad/s)
β	ratio between amplitude of fluctuations and average velocity
ζ	vorticity
ξ	logarithmic radial coordinate, $\ln(r/a)$
θ	angular coordinate
ν	fluid kinematic viscosity
ϕ	dimensionless temperature
ψ	dimensionless stream function.

Subscripts

s	at the surface
∞	at infinite distance from the surface.

relation which applies to the three geometrical shapes was proven to have a maximum deviation of $\pm 15\%$.

The first theoretical investigation of forced convection from an oscillating cylinder was reported by Karanth *et al.* [6] who solved the equations of motion and energy for the two cases of in-line and transverse oscillations for only one value of Reynolds number ($Re = 200$). The average Nusselt number was found to increase with the oscillation in both in-line and transverse directions. Results were compared with previous experimental correlations and excellent agreement was found. In order to trigger vortex shedding, the cylinder was given a small counter-clockwise and then clockwise rotation with a constant angular velocity. However, this effect did not seem to alter the symmetry of the thermal field much. The reported local Nusselt number distributions over a period of one complete cycle for the case of in-line oscillations have an insignificant amount of asymmetry. In addition, the reported isotherm patterns for the same case were also very much symmetric especially in the region adjacent to the cylinder. In fact, vortex shedding in in-line oscillations does not require asymmetry since vortices in this case can be shed in pairs. Another theoretical study on the effect of small stream fluctuations was conducted by Nguyen *et al.* [7] and focused on forced and mixed convection from a rotating cylinder. The study was based on the solution of the vorticity, stream function and energy equations for Reynolds numbers up to 200 in forced convection and 150 in mixed convection and for Grashof numbers up to 2×10^4. The relative speeds of rotation varied from -0.5 to 0.5 and the amplitude of fluctuations was 0.2. In the forced convection case, no vortex shedding occurred for Reynolds numbers up to 200 and the reported streamline pattern was symmetric although symmetry was not imposed in the solution. The symmetry was shown in the reported surface vorticity and pressure distributions for the same case. The effects of Reynolds number, Grashof number, direction of rotation and direction of gravity were discussed in detail but no results were presented on the effect of free-stream fluctuations on the time-average rate of heat transfer.

The effect of acoustic or mechanical transverse oscillations on natural convection from a heated cylinder was studied by Dent [8] who found that cylinder oscillations create a steady streaming motion near the cylinder. A mathematical model was presented for the calculation of the average heat transfer coefficients based on the assumptions that the interaction between the streaming flow and the buoyancy driven flow gives rise to a characteristic velocity that takes both effects into consideration. The model resulted in a correlation that agreed well with the available experimental data provided that the temperature difference is less than 50 F and $Gr > 0.1\ R_{es}^2$ where R_{es} is the streaming Reynolds number. A good survey of the work done on this problem up to the late 60s was reported therein. Heat transfer from a horizontal heated wire performing horizontal oscillations was investigated by Armaly and Madsen [9] who found that an increase in either the amplitude or the frequency of vibration results in increasing heat transfer. A correlation for the average Nusselt number in terms of Reynolds number was reported (Re was based on the average velocity of the reciprocating motion). A similar study was conducted by Kimoto *et al.* [10] who measured both thermal and velocity fields using an interferometer and a hot-wire anemometer, respectively. The study revealed that vibration increases the average heat transfer rate; however, the local heat transfer increases only in the upper and lower parts of the cylinder but decreases on the sides. This decrease on the sides was attributed to a diminishing thermal gradients near the cylinder surface due to boundary-layer separation as well as generation of a small vortex-like turbulence. The effect of vertical vibrations on natural convection from a horizontal cylinder was studied experimentally by Dawood *et al.* [11] who found that the rate of heat transfer can increase by 300% because of vibration effects. It was also found that when the amplitude to diameter ratio (a/D) exceeds 0.5, the increase in heat transfer becomes in linear proportion with a/D irrespective of the frequency or the temperature difference. The effect of vibration on heat transfer was found to be more pronounced when the ratio a/D is greater than 0.25.

This study aims to investigate the effect of free-stream fluctuations on the rate of heat transfer from a straight tube in a forced convection regime in the Reynolds number range up to 500 and Strouhal number range up to π. The flow and thermal fields are assumed to be symmetric based on previous experimental and theoretical results on oscillating and fluctuating flows.

2. CONSERVATION EQUATIONS AND METHOD OF SOLUTION

Consider a straight tube placed with its axis perpendicular to a uniform stream of a cooling fluid of an average velocity, U_o, and a constant temperature, T_∞. The tube has a circular cross-section of radius, a, and an isothermal surface of a constant temperature, T_s. The temperature difference between the tube surface and the oncoming stream is assumed small such that the buoyancy forces have negligible effects on the velocity and thermal fields. The free stream velocity is assumed to be in the form $U = U_o(1+\beta\cos\alpha t)$, where β is the ratio between the amplitude of velocity oscillations and the average free-stream velocity and α is the frequency. The case of a steady free stream represents the special case of $\beta = 0$. The modified (logarithmic) polar coordinate system (ξ, θ) is used such that $\xi = \ln(r/a)$ and the line $\theta = 0$ defines the direction of the free stream. The tube is long enough such that the end effects can be neglected and the flow field is assumed two-dimensional. Neglecting viscous dissipation, the conservation equations can be expressed in the form of vorticity, stream function and energy equations as:

$$e^{2\xi}\frac{\partial \zeta}{\partial t} + \frac{\partial \psi}{\partial \theta}\frac{\partial \zeta}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \zeta}{\partial \theta} = \frac{2}{Re}\left(\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \theta^2}\right) \quad (1)$$

$$e^{2\xi}\zeta = \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \theta^2} \quad (2)$$

$$e^{2\xi}\frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial \theta}\frac{\partial \phi}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \phi}{\partial \theta} = \frac{2}{Pe}\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \theta^2}\right) \quad (3)$$

where Re $(= 2aU_o/v)$ is the Reynolds number, Pe $(= RePr)$ is the Peclet number, Pr is the Prandtl number and v is the kinematic viscosity. All the variables used in the above equations are dimensionless and related to the dimensional quantities (with primes) by

$$t = t'U_o/a,\ \zeta = \zeta' a/U_o,\ \psi = \psi'/aU_o,$$
$$\phi = (T\text{-}T_\infty)/(T_s\text{-}T_\infty).$$

The ξ and θ velocity components are given by

$$v_\xi = \frac{v'_\xi}{U_o} = e^{-\xi}\frac{\partial \psi}{\partial \theta},\quad v_\theta = \frac{v'_\theta}{U_o} = -e^{-\xi}\frac{\partial \psi}{\partial \xi}.$$

The boundary conditions are mainly the no-slip, impermeability and constant temperature on the tube surface and the free stream conditions far away. These can be expressed as:

$$\psi = \frac{\partial \psi}{\partial \xi} = \frac{\partial \psi}{\partial \theta} = 0,\quad \phi = 1,\quad \text{at } \xi = 0 \quad (4a)$$

$$e^{-\xi}\frac{\partial \psi}{\partial \theta} \to (1+\beta\cos St)\cos\theta,$$

$$e^{-\xi}\frac{\partial \psi}{\partial \xi} \to (1+\beta\cos St)\sin\theta,$$

$$\phi \to 0 \quad \text{and} \quad \zeta \to 0 \quad \text{as } \xi \to \infty \quad (4b)$$

where S is the Strouhal number ($S = \alpha a/U_o$). At the start of the computations ($t = 0$), the tube surface temperature is the same as the approaching flow ($\phi = 0$). The velocity boundary layer in the neighbourhood of the tube is first developed with time, t, in the presence of the fluctuating free stream. When the boundary layer becomes thick enough $[(2t/Re)^{1/2} \sim O(1)]$, the tube surface temperature is suddenly increased to T_s (corresponding to $\phi = 1$) and this moment represents the start of the time development of the thermal field. The main mathematical problem is to predict the details of both fields as time increases. The method adopted here is an extension of that used by the present author and reported in Refs. [12, 13]. In this method, the dimensionless stream function, vorticity and temperature are approximated using a finite number of terms in Fourier series and can be expressed as

$$\psi = \sum_{n=1}^{N} f_n(\xi, t)\sin n\theta,\ \zeta = \sum_{n=1}^{N} g_n(\xi, t)\sin n\theta,$$

$$\phi = \frac{1}{2}H_o(\xi, t) + \sum_{n=1}^{N} H_n(\xi, t)\cos n\theta. \quad (5)$$

The use of the above approximations in equations (1)–(3) gives a set of differential equations governing the time variation of the Fourier coefficients (f_n, g_n, H_o and H_n). These can be expressed as:

$$\frac{\partial g_n}{\partial t} = \frac{4}{Re}\left[\frac{\partial^2 g_n}{\partial \xi^2} - n^2 g_n\right] \quad (6a)$$

$$\frac{\partial^2 f_n}{\partial \xi^2} - n^2 f_n = e^{2\xi} g_n \quad (6b)$$

$$e^{2\xi}\frac{\partial H_o}{\partial t} = \frac{2}{Pe}\frac{\partial^2 H_o}{\partial \xi^2} - \sum_{n=1}^{N} n\left(f_n\frac{\partial H_n}{\partial \xi} + H_n\frac{\partial f_n}{\partial \xi}\right) \quad (6c)$$

$$2\,e^{2\xi}\frac{\partial H_n}{\partial t} - \frac{4}{Pe}\left[\frac{\partial^2 H_n}{\partial \xi^2} - n^2 H_n\right] = -nf_n\frac{\partial H_o}{\partial \xi} - \sum_{m=1}^{N}\left\{\frac{\partial H_m}{\partial \xi}[Kf_K + Jf_J] + mH_m\left[\frac{\partial f_K}{\partial \xi} + \mathrm{sgn}(m-n)\frac{\partial f_J}{\partial \xi}\right]\right\} \quad (6d)$$

where $K = m+n$, $J = |m-n|$ and $\mathrm{sgn}(m-n)$ represents the sign of the term $(m-n)$. The conditions given in equations (4a) and (4b) provide the required boundary conditions for all Fourier coefficients. These can be expressed as:

$$f_n = H_n = \frac{\partial f_n}{\partial \xi} = 0 \quad \text{and} \quad H_o = 2 \quad \text{at } \xi = 0 \quad (7a)$$

$$g_n, H_o, H_n \to 0 \quad \text{and}$$
$$f_n \to \delta_n e^{\xi}(1+\beta\cos St) \quad \text{as } \xi \to \infty \quad (7b)$$

where $\delta_n = 1$ for $n = 1$ and $\delta_n = 0$ for $n \neq 1$. By integrating both sides of equation (6b) with respect to ξ between $\xi = 0$ and ∞ and making use of the boundary conditions given in equation (7), one obtains,

$$\int_o^{\infty} e^{(2-n)\xi} g_n(\xi, t)\,d\xi = 2\delta_n(1+\beta\cos St). \quad (8)$$

The above integral condition is similar to that deduced in Ref. [12] though not the same since the boundary conditions are different. It provides means for the calculation of the function g_n at the tube surface ($\xi = 0$) provided that this function is known at all other values of ξ. The local and average Nusselt numbers, Nu and $\overline{Nu}$, are obtained from

$$Nu = \frac{2ah}{k} = -2\left[\frac{\partial \phi}{\partial \xi}\right]_{\xi=0} \quad (9a)$$

$$\overline{Nu} = -\left[\frac{\partial H_o}{\partial \xi}\right]_{\xi=0} \quad (9b)$$

where h is the local heat transfer coefficient. The time-averaged Nusselt number, $\overline{\overline{Nu}}$, is obtained from

$$\overline{\overline{Nu}} = \int_t^{t+T} \overline{Nu}\,dt \quad (9c)$$

where T is the time period for the last complete oscillation.

The numerical scheme used for integrating equations (6a)–(6d) in order to advance the solution of ψ, ζ and ϕ in time is, in principle, the same as that used by the author in Refs. [12, 13] and the details will not be repeated here. The method has been extensively tested and proven to be highly accurate. Table 1 shows a comparison between the obtained values of $\overline{Nu}$ for the case of steady flow and those calculated using the correlations given by Hsu *et al.* as reported in Ref. [14] and by Zijnen [15].

3. DISCUSSION OF RESULTS

The effect of free-stream fluctuations on the velocity and thermal fields in the vicinity of the tube surface is investigated for Reynolds numbers $Re = 50$, 100 and 500 and for Strouhal numbers $S = \pi/4$, $\pi/2$ and π. The relative amplitudes of fluctuations considered are $\beta = 0.2$ and 0.5. The flow and thermal fields are assumed to be symmetric about the line $\theta = 0$. This assumption is based on previous experimental and theoretical studies on oscillating flow over a cylinder, as for example, the works by Williamson [16], Justesen [17] and Badr *et al.* [18]. In these studies, the vortex shedding process occurred in the form of pairs of vortices shedding simultaneously in the wake region without altering the flow field symmetry. This is contrary to the case of a fixed cylinder in which vortices are shed alternatively from above and below the line $\theta = 0$ forming the Karman vortex street. The same phenomenon occurred, though in a different way, in the work by Karanth *et al.* [6] in case of in-line oscillations of a cylinder in a cross stream. In that work, vortex shedding was asymmetrically triggered by a temporary rotation of the cylinder. The isotherm pattern plotted at a large time showed that the flow field symmetry was almost restored. In this work, the symmetry of both flow and thermal fields is imposed in the solution methodology through the Fourier series approximation of ψ, ζ and ϕ.

Table 2 shows the effect of the frequency parameter on the average Nusselt number for various ampli-

Table 1. Comparison between the average Nusselt number obtained in this study and those reported in previous studies in the case of steady free stream

	$\overline{Nu}$				
Re	Present study	Hsu [14]	Zijnen [15]	Gosse [14]	Kramers [14]
20	2.48	2.58	2.59	2.51	2.67
50	3.80	3.82	3.89	3.90	4.00
100	5.44	5.23	5.35	5.45	5.49

Table 2. The effect of amplitude and frequency of free-stream fluctuations on the time-averaged Nusselt number

Case no.	Reynolds number (Re)	Amplitude (β)	Strouhal number (S)	$\overline{\overline{Nu}}$	$\overline{Nu}$ for steady stream ($\beta = S = 0$)
1-a			$\pi/4$	3.82	3.80
1-b	50	0.2	$\pi/2$	3.75	
1-c			π	3.72	
1-d			$\pi/4$	3.89	
1-e	50	0.5	$\pi/2$	3.79	
1-f			π	3.76	
2-a			$\pi/4$	5.59	5.44
2-b	100	0.2	$\pi/2$	5.50	
2-c			π	5.46	
2-d			$\pi/4$	5.91	
2-e	100	0.5	$\pi/2$	5.62	
2-f			π	5.47	
3-a			$\pi/4$	16.77	15.95*
3-b	500	0.2	$\pi/2$	16.27	
3-c			π	16.16	
3-d			$\pi/4$	19.30	
3-e	500	0.5	$\pi/2$	18.42	
3-f			π	16.83	

* The quasi-steady state has not been reached and the values reported for $\overline{Nu}$ and $\overline{\overline{Nu}}$ correspond to the last cycle (for details see Fig. 2).

tudes. In the low Reynolds number case of $Re = 50$, the rate of heat transfer may slightly decrease below that of steady free stream due to the effect of main stream fluctuations. This phenomenon occurred for the two amplitudes of $\beta = 0.2$ and 0.5 with more reduction in $\overline{\overline{Nu}}$ with increasing frequency. The same phenomenon was reported by Zijnen [1] for low Reynolds number flow ($Re < 5$). The other phenomenon reported in the experimental work by Sreenivasan and Ramachandran [2] was the insignificant change in the rate of heat transfer for vibrational velocities up to 20% of the flow velocity. This phenomenon is also shown in Table 2 since the maximum change in $\overline{\overline{Nu}}$ due to the effect of free-stream fluctuations in case of $\beta = 0.2$ is 2.1% for $Re = 50$, 2.7% for $Re = 100$ and 4.9% for $Re = 500$. These changes are very small and may lie within the uncertainty limits of experimental measurements. In these Reynolds number cases, the average Nusselt number decreases with the increase of the Strouhal number for a constant amplitude.

The numerical values of $\overline{\overline{Nu}}$ for the two amplitudes of fluctuation are given in Table 2 for every value of the Strouhal number considered. In the low Reynolds number case of $Re = 50$, increasing the amplitude from $\beta = 0.2$ to $\beta = 0.5$ results in a small increase in $\overline{\overline{Nu}}$. In comparison with the case of steady free stream, the low Strouhal number fluctuations ($S = \pi/4$) resulted in a small increase in $\overline{\overline{Nu}}$ (2.37%); however, further increase of S tends to decrease $\overline{\overline{Nu}}$ to values below that of no fluctuations. The two higher Re cases of 100 and 500 show that the effect of fluctuations becomes more pronounced with increasing Re. Within the range of variables considered, the maximum increase of $\overline{\overline{Nu}}$ due to fluctuations occurs when $\beta = 0.5$ and $S = \pi/4$ and reached 8.6% for $Re = 100$ and 21% for $Re = 500$. This increase becomes less at higher values of the Strouhal number. One should emphasize here that the periodic variation has been achieved for the cases of $Re = 50$ and 100; however, for the case of $Re = 500$, the thermal field was still developing for all values of S at the time when computations were terminated. Figure 1 shows the effect of free-stream fluctuations on $\overline{Nu}$ for the case of $Re = 50$ during the time period $t = 16$–24 which corresponds to one complete cycle for $S = \pi/4$ and four complete cycles for $S = \pi$. The time-variation of the free-stream velocity is also shown in each graph. It is clear from the figure that the $\overline{Nu}$ fluctuations have a phase lag behind the free-stream velocity fluctuations. Figure 2 shows the time variation of $\overline{Nu}$ for different values of β and S in the case of $Re = 500$. The number of peaks in the $\overline{Nu}$-t curves is the same as that in the U-t curves for all values of β and S except for $S = \pi/4$. In that case, vortex shedding created one additional peak.

The effect of amplitude of fluctuations on the local Nusselt number distribution for the case of $Re = 50$ and $S = \pi/4$ is shown in Fig. 3(a, b) for half of the tube surface ($\theta = 0$–180°C where $\theta = 0$ defines the rear stagnation point). Figure 3(a) corresponds to maximum free-stream velocity, U_{max}, and Fig. 3(b) corresponds to minimum velocity, U_{min}. In Fig. 3(a), Nu is higher than its steady value on almost all the tube surface and this is expected since higher stream velocities lead to higher velocity and temperature gradients. The figure also shows that the increase of the amplitude, β, results in further increase of Nu over all the tube surface. However, when the free-stream velocity is minimum, Nu decreases below its steady value all over the tube surface except for the rear part ($\theta \approx 0$–$60°$) where Nu becomes higher as shown in Fig. 3(b). In order to explain this phenomenon, the surface vorticity distributions are plotted for the same two cases in Fig. 4(a, b). Figure 4(a) shows a small value of $|\zeta_s|$ in the wake region ($\theta \approx 0$–$60°$) indicating

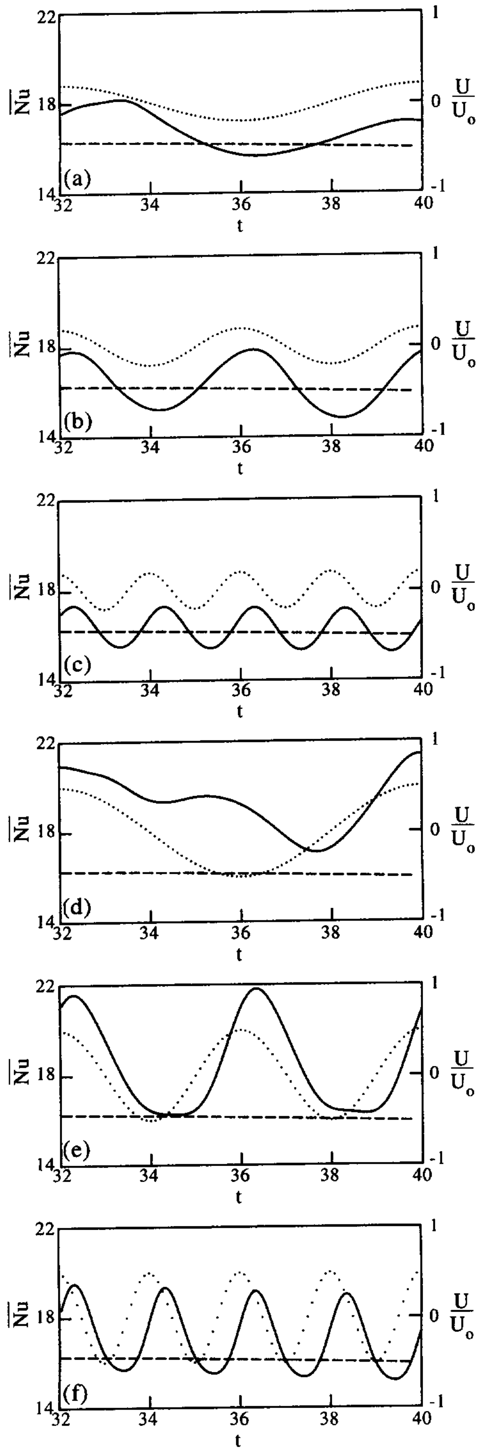

Fig. 1. The effect of free-stream fluctuations on the average Nusselt number during the time period $t = 16$–24 for the cases 1-a, 1-b, 1-c, 1-d, 1-e and 1-f: (–) $\overline{Nu}$ for fluctuating flow; (---) $\overline{Nu}$ for steady free stream; (. . .) variation of U/U_o (for details see Table 2).

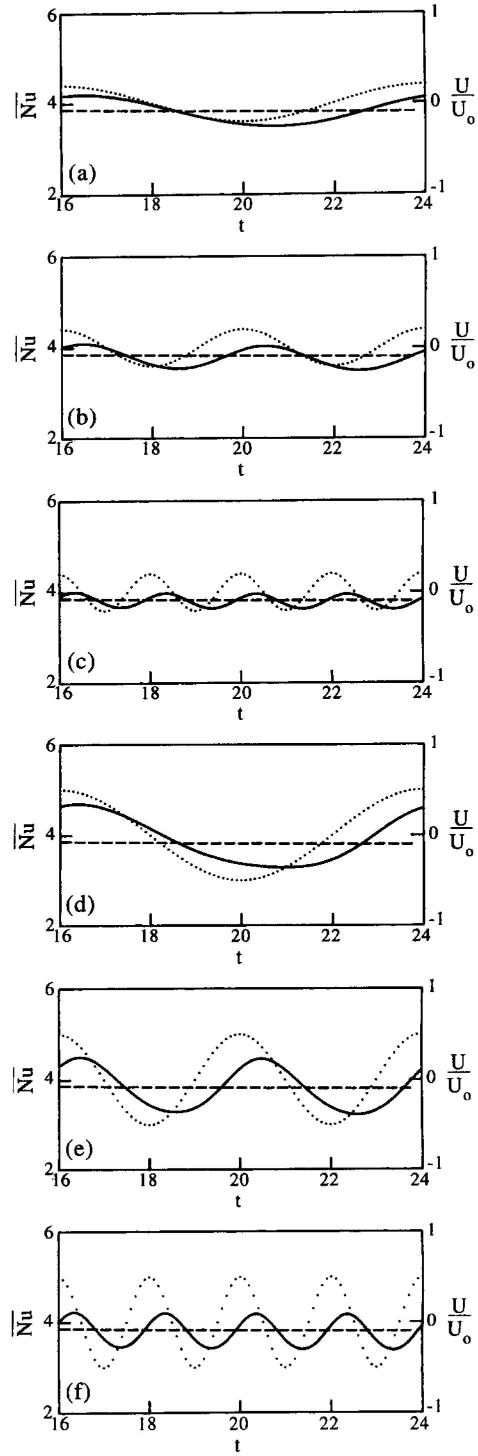

Fig. 2. The effect of free-stream fluctuations on the average Nusselt number during the time period $t = 32$–40 for the cases 3-a, 3-b, 3-c, 3-d, 3-e and 3-f: (–) $\overline{Nu}$ for fluctuating flow; (---) $\overline{Nu}$ for steady free stream; (. . .) variation of U/U_o (for details see Table 2).

a small velocity gradient and accordingly a weak vortical motion when $U = U_{max}$. On the other hand, when $U = U_{min}$, the size and strength of the vortex-driven flow in the wake region becomes more prominent as shown in Fig. 4b. For example, in case of $\beta = 0.5$, the wake behind the tube occupies the region between $\theta \approx -90°$ and $\theta \approx 90°$ with higher value of $|\zeta_s|$ in comparison with the steady case. Stronger vortical

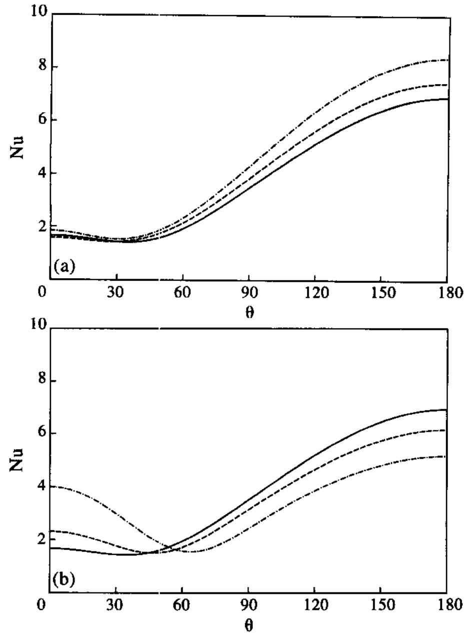

Fig. 3. The effect of amplitude of fluctuations on the local Nusselt number distribution for the case of $Re = 50$, $S = \pi/4$ when (a) $U = U_{max}$; and (b) $U = U_{min}$: (---) $\beta = 0.2$; (-··-) $\beta = 0.5$; (–) $\beta = 0$.

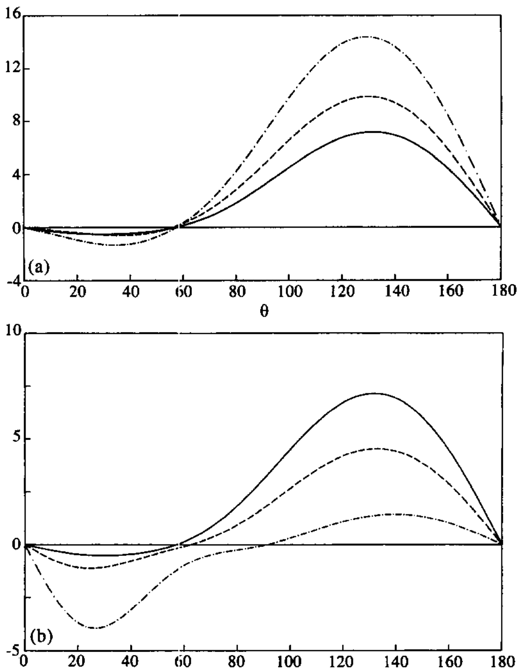

Fig. 4. The effect of amplitude of fluctuations on the surface vorticity distribution for the case of $Re = 50$, $S = \pi/4$ when (a) $U = U_{max}$; and (b) $U = U_{min}$: (---) $\beta = 0.2$; (-··-) $\beta = 0.5$; (–) $\beta = 0$.

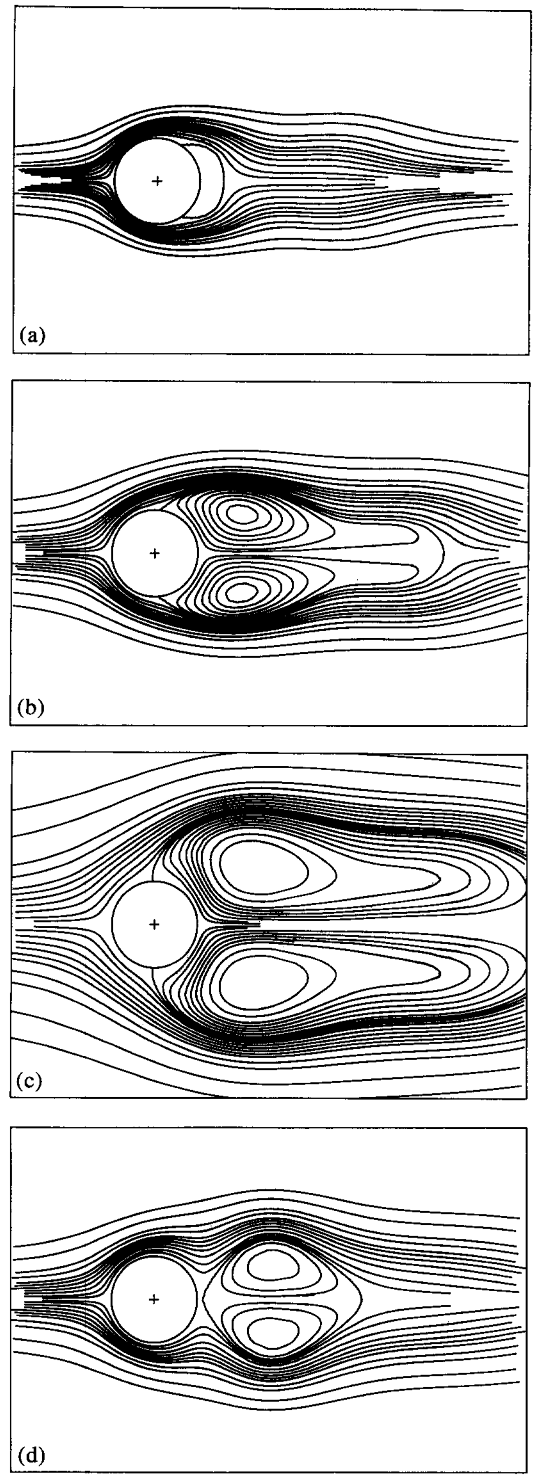

Fig. 5. The time variation of the streamline pattern plotted every one-quarter cycle for the case of $Re = 50$, $S = \pi/4$ and $\beta = 0.5$: (a) $t = 16$; (b) $t = 18$; (c) $t = 20$; and (d) $t = 22$.

motion in the wake region leads to higher velocity and temperature gradients and accordingly higher rate of heat transfer. In order to see the details of the velocity and thermal fields corresponding to U_{min} and U_{max}, the streamline and isotherm contours describing the changes during one complete cycle are plotted in Figs. 5 and 6 for the case of $Re = 50$, $S = \pi/4$ and $\beta = 0.5$. Figures 5(a) and 6(a) show the streamline and isotherm patterns corresponding to maximum free-stream velocity, while Figs. 5(c) and 6(c) correspond

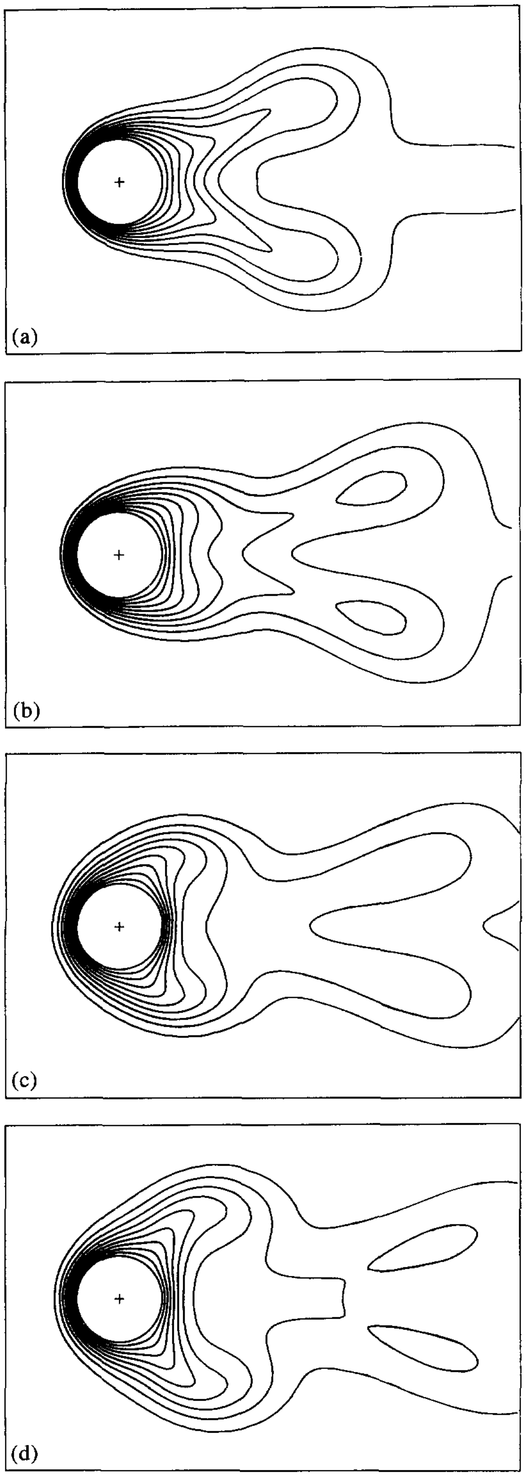

Fig. 6. The time variation of the isotherm pattern plotted every one-quarter cycle for the case of $Re = 50$, $S = \pi/4$ and $\beta = 0.5$: (a) $t = 16$; (b) $t = 18$; (c) $t = 20$; and (d) $t = 22$. Isotherms plotted are $\phi = 0.1$ (0.1) 0.9.

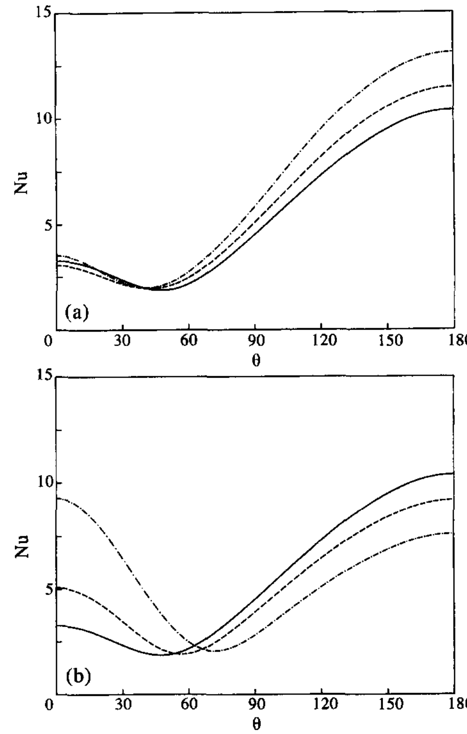

Fig. 7. The effect of amplitude of fluctuations on the local Nusselt number distribution for the case of $Re = 100$, $S = \pi/4$ when (a) $U = U_{max}$; and (b) $U = U_{min}$: (---) $\beta = 0.2$; (-·-) $\beta = 0.5$; (–) $\beta = 0$.

to the minimum free-stream velocity. It is clear that the size of the wake region is greater and the enclosed vortical motion is stronger when $U = U_{min}$. The closer isotherms in the wake region in Fig. 6(c) in comparison with those in Fig. 6(a) explain clearly the same phenomenon. The local Nusselt number distributions for the case of $Re = 100$, $S = \pi/4$ are shown in Fig. 7(a) for $U = U_{max}$ and in Fig. 7b for $U = U_{min}$. The trend is very much the same as that discussed in the case of $Re = 50$.

The variation of the streamline pattern for the case 3d ($Re = 500$, $\beta = 0.5$ and $S = \pi/4$) during one complete cycle is shown in Fig. 8. Figure 8(a) (corresponding to $t = 32$ at which $U = U_{max}$) shows the start of the formation of two counter-rotating vortices in the wake near the tube surface while the shape of the streamlines downstream shows the traces of two decayed pairs of vortices. At $t = 34$ (corresponding to $U = U_o$), this pair of vortices is shed away while another pair has been formed as shown in Fig. 8(b). The vortex shedding process occurs here in pairs similar to what was found experimentally by Williamson [16] in oscillating flows. The growth of the two pairs of vortices continues during flow deceleration until reaching $t = 36$ at which $U = U_{min}$ as shown in Fig. 8(c). Figure 8(d) shows the situation at $t = 38$ at which $U = U_o$. The accelerating flow (between $t = 36$ and $t = 38$) causes the two pairs of vortices to decrease in size, but with increasing strength. The point of separation moved from $\theta = 125°$ in Fig. 8(c) to $\theta = 26°$ in Fig. 8(d) indicating the beginning of the shedding process. Figure 9(a)–(d) show the variation of the isotherm patterns for the same complete cycle.

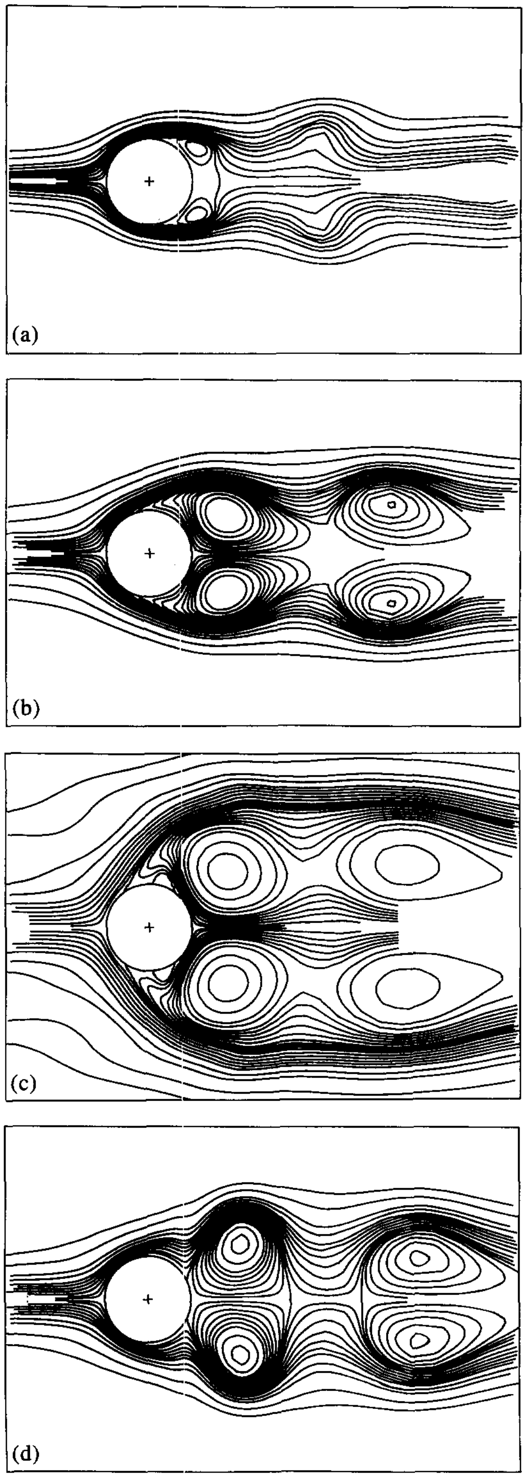

Fig. 8. The time variation of the streamline pattern plotted every one-quarter cycle for the case of $Re = 500$, $S = \pi/4$ and $\beta = 0.5$: (a) $t = 32$; (b) $t = 34$; (c) $t = 36$; and (d) $t = 38$.

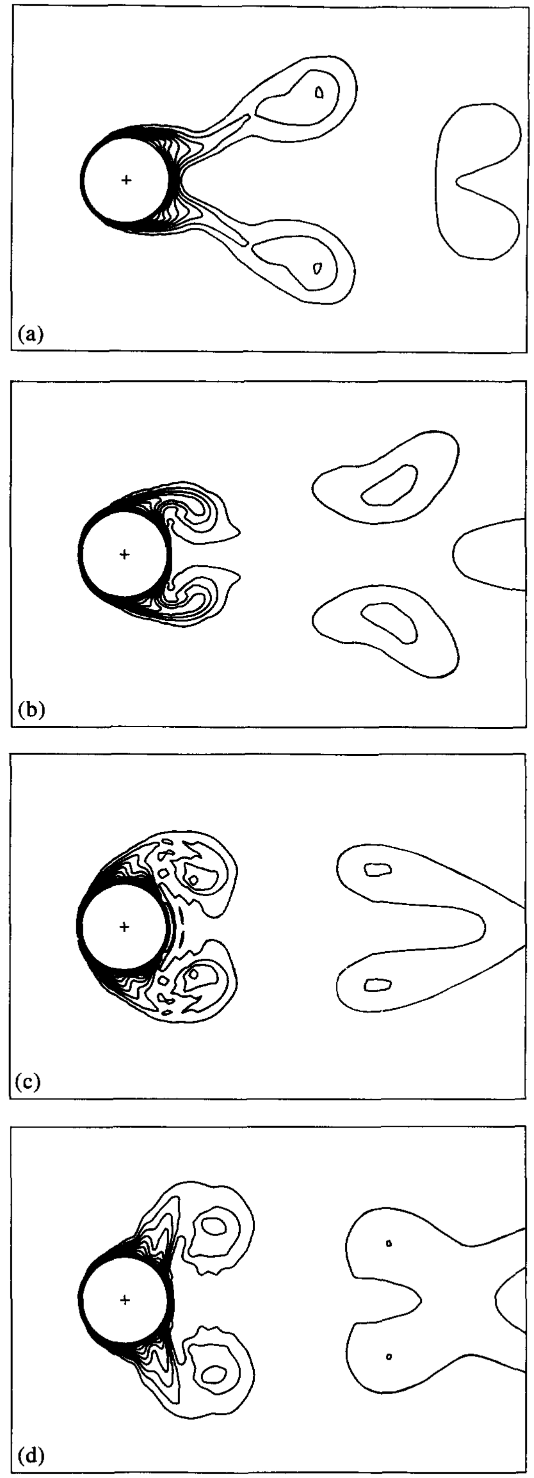

Fig. 9. The time variation of the isotherm pattern plotted every one-quarter cycle for the case of $Re = 500$, $S = \pi/4$ and $\beta = 0.5$: (a) $t = 32$; (b) $t = 34$; (c) $t = 36$; and (d) $t = 38$. Isotherms plotted are $\phi = 0.1\ (0.1)\ 0.9$.

4. CONCLUSIONS

The effect of free-stream fluctuation on forced convection from a tube of circular cross-section is investigated in the Reynolds number range $Re = 50$–500 and the Strouhal number range $S = \pi/4$ to π. The study revealed that the changes in the time-averaged Nusselt number, $\overline{\overline{Nu}}$, due to fluctuations are small in the low Reynolds number case of $Re = 50$. At this

Reynolds number, the free-stream fluctuations may result in either an increase or a decrease in $\overline{\overline{Nu}}$ depending on the amplitude and frequency of fluctuations. However, at higher Re values, the effect of fluctuations becomes significant and $\overline{\overline{Nu}}$ increases with the increase of the amplitude and slightly decreases with the increase of frequency. Vortex shedding is shown to occur in pairs in a prescribed symmetrical velocity field similar to what was found experimentally in oscillating flows.

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